Cosmic Inflation

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1 Overview of Inflation

This section is almost verbatim (minor changes) from https://wmap.gsfc.nasa.gov/universe/bb_cosmo_infl.html

1.1 What is the Inflation Theory?

Inflation Theory proposes a period of extremely rapid (exponential) expansion of the universe during its first few moments. It was developed around 1980 to explain several puzzles with the standard Big Bang theory, in which the universe expands relatively gradually throughout its history.

1.2 Limitations of the Big Bang Theory

While the Big Bang theory successfully explains the "blackbody spectrum" of the cosmic microwave background radiation and the origin of the light elements, it has three significant problems:

- The Flatness Problem:
  WMAP has determined the geometry of the universe to be nearly flat, which means mass-energy density must be finely tuned to critical mass-energy density. A universe as flat as we see it today would require such extreme fine-tuning of conditions in the past, which would be an unbelievable coincidence.
- The Horizon Problem:
  Distant regions of space in opposite directions of the sky are so far apart that, assuming standard Big Bang expansion, they could never have been in causal contact with each other. This is because the light travel time between them exceeds the age of the universe. Yet the uniformity of the cosmic microwave background temperature tells us that these regions must have been in contact with each other in the past.
- The Monopole Problem:
  Big Bang cosmology (from GUT theories, primarily) predicts that a very large number of heavy, stable "magnetic monopoles" should have been produced in the early universe. However, magnetic monopoles have never been observed, so if they exist at all, they are much more rare than the Big Bang theory predicts.

1.3 The Inflation Theory

The Inflation Theory, developed by Alan Guth, Andrei Linde, and Alexei Starobinsky, Paul Steinhardt, and Andy Albrecht, offers solutions to these problems and several other open questions in cosmology. It proposes a period of extremely rapid (exponential) expansion of the universe prior to the more gradual Big Bang expansion, during which time the energy density of the universe was dominated by a cosmological constant-type of vacuum energy that later decayed to produce the matter and radiation that fill the universe today.

Inflation was both rapid, and strong. It increased the linear size of the universe by more than 60 "e-folds", or a factor of $\sim 10^{26}$ in $\sim 10^{-32}$ seconds! It grew from being 10^{-26}m in diameter (100 billion times smaller than a proton) to about the size of a grapefruit in that time. At that point on, the universe stopped expanding exponentially and began behaving more in a manner consistent with dynamics we are more familiar with. It grew rapidly, but at a rate nothing like that during inflation. Many call that moment, when exponential expansion turned off, the Big Bang.

Inflation is now considered an extension of the Big Bang theory since it explains the above puzzles so well, while retaining the basic paradigm of a homogeneous expanding universe. Moreover, Inflation Theory links important ideas in modern physics, such as symmetry breaking and phase transitions, to cosmology.

Note that inflation involves a growth in space itself and so, can, and does (in theory), occur at speeds far greater than light speed. Space can stretch at rates exceeding c, but objects within that space cannot move faster than c with respect to that space.

1.4 How Does Inflation Solve these Problems?

- The Flatness Problem:
  Imagine living on the surface of a soccer ball (a 2-dimensional world). It might be obvious to you that this surface was curved and that you were living in a closed universe. However, if that ball expanded to the size of the Earth, it would appear flat to you, even though it is still a sphere on larger scales. Now imagine increasing the size of that ball to astronomical scales. To you, it would appear to be flat as far as you could see, even though it might have been very curved to start with. Inflation stretches any initial curvature of the 3-dimensional universe to near flatness.
- The Horizon Problem:
  Since Inflation supposes a burst of exponential expansion in the early universe, it follows that distant regions were

actually much closer together prior to Inflation than they would have been with only standard Big Bang expansion. Thus, such regions could have been in causal contact prior to Inflation and could have attained a uniform temperature.

- The Monopole Problem:
  Inflation allows for magnetic monopoles to exist as long as they were produced prior to the period of inflation. During inflation, the density of monopoles drops exponentially, so their abundance drops to undetectable levels.

As a bonus, Inflation also explains the origin of structure in the universe. Prior to inflation, the portion of the universe we can observe today was microscopic, and quantum fluctuation in the density of matter on these microscopic scales expanded to astronomical scales during Inflation. Over the next several hundred million years, the higher density regions condensed into stars, galaxies, and clusters of galaxies.

2 Technical Background

2.1 Scalar QFT Field Equation in Classical GR Spacetime

We can express the action S in terms of physical quantities (those we would measure with instruments in the world) or coordinate quantities (using generalized coordinates)

$$S=\int \underbrace{\hat{\mathcal{L}}}_{\substack{\text{phys}\\ \text{Lagr}}} \underbrace{\sqrt{-g}\,d^4x}_{\text{phys }d\underset{\sim}{V}} = \int \underbrace{\mathcal{L}}_{\substack{\text{coord}\\ \text{Lagr}}} \underbrace{d^4x}_{\substack{\text{coord}\\ dV}} \qquad \rightarrow \qquad \mathcal{L}=\sqrt{-g}\;\hat{\mathcal{L}}\,. \tag{1}$$

In flat space QFT, for a real massless scalar, we have a physical Lagrangian (see Klauber, Vol. 2, pg. 163, (6-2) for a free scalar, where the potential there is simply a mass squared term, but here it is replaced by a potential density)

$$\hat{\mathcal{L}}_\phi=\tfrac{1}{2}\phi^{,\mu}\phi_{,\nu}-\mathcal{V}(\phi)=\tfrac{1}{2}\eta_{\mu\nu}\phi^{,\mu}\phi^{,\nu}-\mathcal{V}(\phi) \qquad \eta_{\mu\nu}=\text{ Minkowski metric}\,, \tag{2}$$

In going to a curved space, we take $\eta_{\mu\nu}\rightarrow g_{\mu\nu}$. It is also easier dealing with the coordinate Lagrangian, as the Euler-Lagrange equation is derived from it in the usual way, so we can simply assume that equation is valid for the coordinate Lagrangian. Thus, combining (1) and (2), we have

$$\hat{\mathcal{L}}_\phi=\tfrac{1}{2}g_{\mu\nu}\phi^{,\mu}\phi^{,\nu}-\mathcal{V}(\phi)=\tfrac{1}{2}g^{\mu\nu}\phi_{,\mu}\phi_{,\nu}-\mathcal{V}(\phi) \qquad \rightarrow \qquad \mathcal{L}_\phi=\sqrt{-g}\left(\tfrac{1}{2}g^{\mu\nu}\phi_{,\mu}\phi_{,\nu}-\mathcal{V}(\phi)\right). \tag{3}$$

With (3) into the Euler-Lagrange equation,

$$\partial_\alpha\frac{\partial\mathcal{L}_\phi}{\partial\phi_{,\alpha}}-\frac{\partial\mathcal{L}_\phi}{\partial\phi}=0 \tag{4}$$

we have

$$\begin{aligned}
0&=\partial_\alpha\sqrt{-g}\,\frac{\partial}{\partial\phi_{,\alpha}}\left(\tfrac{1}{2}g^{\mu\nu}\phi_{,\mu}\phi_{,\nu}-\mathcal{V}\right)-\sqrt{-g}\,\frac{\partial}{\partial\phi}\left(\tfrac{1}{2}g^{\mu\nu}\phi_{,\mu}\phi_{,\nu}-\mathcal{V}\right)\\
&=\partial_\alpha\tfrac{1}{2}g^{\mu\nu}\sqrt{-g}\left(\left(\frac{\partial}{\partial\phi_{,\alpha}}\phi_{,\mu}\right)\phi_{,\nu}+\phi_{,\mu}\frac{\partial}{\partial\phi_{,\alpha}}\phi_{,\nu}\right)+\sqrt{-g}\,\frac{\partial\mathcal{V}}{\partial\phi}\\
&=\partial_\alpha\tfrac{1}{2}g^{\mu\nu}\sqrt{-g}\left(\delta^\alpha_\mu\phi_{,\nu}+\phi_{,\mu}\delta^\alpha_\nu\right)+\sqrt{-g}\,\frac{\partial\mathcal{V}}{\partial\phi}=\partial_\alpha\sqrt{-g}\,\tfrac{1}{2}\left(g^{\alpha\nu}\phi_{,\nu}+g^{\mu\alpha}\phi_{,\mu}\right)+\sqrt{-g}\,\frac{\partial\mathcal{V}}{\partial\phi}\\
&=\partial_\alpha\sqrt{-g}\,\tfrac{1}{2}\left(g^{\alpha\beta}\phi_{,\beta}+g^{\beta\alpha}\phi_{,\beta}\right)+\sqrt{-g}\,\frac{\partial\mathcal{V}}{\partial\phi}=\partial_\alpha\sqrt{-g}\left(g^{\alpha\beta}\phi_{,\beta}\right)+\sqrt{-g}\,\frac{\partial\mathcal{V}}{\partial\phi}
\end{aligned} \tag{5}$$

or, finally, as our real scalar field equation for a curved spacetime background.

$$\boxed{\frac{1}{\sqrt{-g}}\partial_\alpha\sqrt{-g}\left(g^{\alpha\beta}\phi_{,\beta}\right)+\frac{\partial\mathcal{V}}{\partial\phi}=0} \tag{6}$$

Note that in Minkowski (flat 4D) space, $g^{\alpha\beta}=\eta^{\alpha\beta}=$ constant matrix, so (6) reduces in that case to the field equation we are familiar with for a real scalar.

2.2 Cosmology Equations

2.2.1 The Local Energy Balance Equation

$$\boxed{\dot{\rho} = -3(\rho + p)\frac{\dot{a}}{a} = -3(\rho + p)H} \tag{7}$$

2.2.2 The First Friedmann Equation

We cast any cosmological constant as a vacuum contribution to the stress-energy tensor, so ρ here includes that.

$$\boxed{\left(\frac{\dot{a}}{a}\right)^2 = H^2 = \frac{8\pi G\rho}{3} - k\left(\frac{a_0}{a}\right)^2} \tag{8}$$

Note, in passing, that the second Friedmann equation is not independent of (7) and (8), as it can be derived from combining them. So, we can use any two of the three equations in any analysis. In any particular case, we choose the two that make analysis simplest. For us, in this document, these are (7) and (8).

2.2.3 The Robertson-Walker/Friedmann Metric

In the R-W/Friedman (isotropic and homogeneous) universe, the line element is, where S_k varies for flat, positive, and negative 3D space, and where we use the QFT metric signature (negative spatial components) of (+,−,−,−), not the usual relativity theory version (negative time component) of (−, +,+,+),

$$ds^2 = c^2dt^2 - a^2(t)\left(dr^2 + S_k^2\left(d\theta^2 + sin^2\theta d\phi^2\right)\right), \tag{9}$$

and

$$S_k^2 = sin^2 r \quad k = 1\ (\text{positive curvature}) \qquad S_k^2 = r^2 \quad k = 0\ (\text{flat}) \qquad S_k^2 = sinh^2 r \quad k = -1\ (\text{negative curvature}) \tag{10}$$

Thus, the metric is

$$\boxed{g_{\mu\nu} = \begin{bmatrix} c^2 & & & \\ & -a^2(t) & & \\ & & -a^2(t)S_k^2 & \\ & & & -a^2(t)S_k^2 sin^2\theta \end{bmatrix}}. \tag{11}$$

2.3 Energy Density and Pressure for a Real Scalar Field

2.3.1 Energy Density

Note that ρ is energy density, but expressed for a field, that is simply $\mathcal{H}$. From Klauber, Vol. 1, pg 156, Wholeness Chart 5-4, 1st column, 6th row, not counting title boxes, we find $\mathcal{H}$ for the free complex scalar field with a mass term for the potential. Note from (2) (and from Klauber, Vol. 2, pg. 163, comparing (6-1) to (6-2)) that we have a factor of ½ in the expressions for $\mathcal{L}$ and $\mathcal{H}$ for real scalars, that we don't have for complex ones.

Thus, from those references,

$$\mathcal{H} = \boxed{\rho_\phi = \frac{\dot{\phi}^2}{2} + \frac{(\nabla\phi)^2}{2} + \mathcal{V}} \tag{12}$$

2.3.2 Pressure

To start, we repeat (2) here, for convenience,

$$\hat{\mathcal{L}}_\phi = \tfrac{1}{2}\phi^{,\mu}\phi_{,\nu} - \mathcal{V}(\phi) = \tfrac{1}{2}\eta_{\mu\nu}\phi^{,\mu}\phi^{,\nu} - \mathcal{V}(\phi) \qquad \eta_{\mu\nu} = \text{ Minkowski metric} \qquad \text{repeat of (2)}$$

The known expression for the stress-energy tensor for a perfect fluid, with physical (what would be measured with instruments) components measured in the rest frame of the fluid, is

$$\hat{T}^{\mu\nu} = \begin{bmatrix} \rho_\phi & & & \\ & p_\phi & & \\ & & p_\phi & \\ & & & p_\phi \end{bmatrix} = \begin{bmatrix} \hat{T}^{00} & & & \\ & \hat{T}^{11} & & \\ & & \hat{T}^{22} & \\ & & & \hat{T}^{33} \end{bmatrix}, \tag{13}$$

and the relation we already stated for ρ_ϕ, (12), with alternative notation on the RHS, is

$$\rho_\phi = \hat{T}^{00} = \frac{\dot{\phi}^2}{2} + \frac{(\nabla\phi)^2}{2} + \mathcal{V} = \tfrac{1}{2}\left(\partial^0\phi\right)\left(\partial^0\phi\right) + \tfrac{1}{2}\left(\partial^i\phi\right)\left(\partial^i\phi\right) + \mathcal{V}\,. \tag{14}$$

Note that (14) can be expressed as

$$\hat{T}^{00} = \left(\partial^0\phi\right)\left(\partial^0\phi\right) - \eta^{00}\hat{\mathcal{L}} \tag{15}$$

H.W. Problem #1. Deduce (15).

From (15), which is a relation of particular tensor components, we can deduce the general form for all components,

$$\hat{T}^{\mu\nu} = \left(\partial^\mu\phi\right)\left(\partial^\nu\phi\right) - \eta^{\mu\nu}\hat{\mathcal{L}}\,. \tag{16}$$

From (13) and (16),

$$\begin{aligned} p_\phi = \hat{T}^{11} &= \left(\partial^1\phi\right)\left(\partial^1\phi\right) - \eta^{11}\hat{\mathcal{L}} = \left(\partial^1\phi\right)\left(\partial^1\phi\right) - (-1)\hat{\mathcal{L}} = \left(\frac{\partial\phi}{\partial x^1}\right)^2 + \tfrac{1}{2}\eta_{\mu\nu}\phi^{,\mu}\phi^{,\nu} - \mathcal{V}(\phi) \\ &= \left(\frac{\partial\phi}{\partial x^1}\right)^2 + \tfrac{1}{2}\phi^{,0}\phi^{,0} - \tfrac{1}{2}\phi^{,i}\phi^{,i} - \mathcal{V}(\phi). \end{aligned} \tag{17}$$

Thus, the pressure in the x^1 direction, which equals the pressure in any direction, is

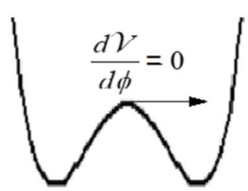
$$\boxed{p_{\phi 1} = \left(\frac{\partial\phi}{\partial x^1}\right)^2 + \frac{\dot{\phi}^2}{2} - \frac{(\nabla\phi)^2}{2} - \mathcal{V}} = \text{pressure in any direction.} \tag{18}$$

3 Assumptions

For the inflation period, we make the following reasonable assumptions.

1. Space is close enough to flat that we can take $k = 0$ in (8), and $S_k = r$ in (11). For non-flat spaces, for initial growth leading quickly to $a >> a_0$, the last term in (8) will be effectively zero anyway. This implies a metric approaching that of a flat space as a increases. And since $\sin^2$ for positive curvature varies between 0 and 1, the last two non-zero components in (11) will be on the order of $a(t)$, and thus, (11) can be considered equal to an approximation of the metric for flat space.

2. $\dfrac{d\mathcal{V}}{d\phi}$ is small, and thus, negligible in (6). This is reasonable, as the inflaton field (ϕ here for us) is presumed to have a potential much like the Higgs. That is, for it, the universe is presumed to slide off the unstable equilibrium at the



top of the Mexican hat and seek a lower, stable potential level. At the top, $\dfrac{d\mathcal{V}}{d\phi} = 0$, and even in the first part of the "slide", this derivative remains close to zero. Later, if the other term in (6) grows large, this term will be dwarfed.

3. $\dfrac{\partial \phi}{\partial t} = \dot{\phi}$ is initially small
4. $\nabla \phi$ is small and negligible in (6), (12), and (18). The field changes little over spatial distances.
5. Consider it is possible that $\mathcal{V} \gg \dfrac{\dot{\phi}^2}{2}$ (P.E. is much greater than K.E.), so we can ignore the time derivatives in (12) and (18).
6. ρ_ϕ >> energy density of all other fields/particles.

4 Resulting Key Equations

For flat, or effectively flat, space, (11) becomes, in polar coordinates,

$$g_{\mu\nu} = \begin{bmatrix} 1 & & & \\ & -a^2(t) & & \\ & & -a^2(t)r^2 & \\ & & & -a^2(t)r^2 \sin^2\theta \end{bmatrix}. \tag{19}$$

The same space in orthogonal coordinates has

$$g_{\mu\nu} = \begin{bmatrix} 1 & & & \\ & -a^2(t) & & \\ & & -a^2(t) & \\ & & & -a^2(t) \end{bmatrix}, \tag{20}$$

so,

$$\sqrt{-g} = \sqrt{-Det\, g} = a^3(t). \tag{21}$$

Using (21) in (6), with assumption 2, we have the field equation for the inflaton,

$$\frac{1}{a^3(t)} \frac{\partial}{\partial x^\alpha}\left(a^3(t)\left(g^{\alpha\beta}\phi_{,\beta}\right)\right) + \frac{\partial \mathcal{V}}{\partial \phi} = 0. \tag{22}$$

With assumption 4, we can ignore space derivatives in (22), so it becomes

$$\begin{aligned} 0 &= \frac{1}{a^3(t)} \frac{\partial}{\partial t}\left(a^3(t)\left(g^{\alpha 0}\phi_{,0}\right)\right) + \frac{\partial \mathcal{V}}{\partial \phi} = \frac{1}{a^3(t)} \frac{\partial}{\partial t}\left(a^3(t)\left(g^{00}\phi_{,0}\right)\right) + \frac{\partial \mathcal{V}}{\partial \phi} = \frac{1}{a^3}\frac{\partial}{\partial t}\left(a^3\phi_{,0}\right) + \frac{\partial \mathcal{V}}{\partial \phi} \\ &= \frac{1}{a^3}\frac{\partial}{\partial t}\left(a^3\dot{\phi}\right) + \frac{\partial \mathcal{V}}{\partial \phi} = \frac{1}{a^3} a^3 \frac{\partial \dot{\phi}}{\partial t} + \frac{1}{a^3}\dot{\phi}\frac{\partial a^3}{\partial t} + \frac{\partial \mathcal{V}}{\partial \phi} = \ddot{\phi} + \frac{3}{a^3}a^2 \dot{a}\dot{\phi} + \frac{\partial \mathcal{V}}{\partial \phi}. \end{aligned} \tag{23}$$

Or, finally,

$$\ddot{\phi} + 3\left(\frac{\dot{a}}{a}\right)\dot{\phi} = -\frac{\partial \mathcal{V}}{\partial \phi}. \tag{24}$$

With assumption 2, we have

$$\ddot{\phi} + 3\left(\frac{\dot{a}}{a}\right)\dot{\phi} = 0. \tag{25}$$

From (25), and our assumption 3 (the initial time derivative of the inflation $\dot{\phi}$ is not too large), then $\ddot{\phi}$ is not large, so $\dot{\phi}$ will not grow rapidly. This justifies assumption 5, that the potential is much larger than the kinetic term for ϕ throughout the inflation process.

With that assumption, the energy density and pressure relations of (12) and (18) become

$$\rho_\phi \approx \mathcal{V} \qquad p_\phi \approx -\mathcal{V}. \tag{26}$$

With (26), the time rate of change of mass-energy density (7) is

$$\dot{\rho} = -3(\rho + p)\frac{\dot{a}}{a} \approx -3(\mathcal{V} - \mathcal{V})\frac{\dot{a}}{a} = 0\,, \tag{27}$$

and the mass-energy density remains effectively constant

$$\rho_\phi = \text{ constant}\,, \tag{28}$$

so, the stress-energy tensor becomes

$$T_\phi^{\mu\nu} = \begin{bmatrix} \rho_\phi & & & \\ & p_\phi & & \\ & & p_\phi & \\ & & & p_\phi \end{bmatrix} = \begin{bmatrix} \mathcal{V} & & & \\ & -\mathcal{V} & & \\ & & -\mathcal{V} & \\ & & & -\mathcal{V} \end{bmatrix} = \eta^{\mu\nu}\mathcal{V}\,. \tag{29}$$

We should recall from cosmology that a constant stress-energy tensor of form (29) can be interpreted alternatively as a cosmological constant in the 1st Friedmann equation (8). Here we continue to keep it as a stress-energy tensor, however, so we use $\rho_\phi = \mathcal{V}$ = constant for ρ in that equation. Thus, (8) reduces, with k taken = 0, to

$$\left(\frac{\dot{a}}{a}\right)^2 = H_{\mathcal{V}}^2 = \frac{8\pi G\mathcal{V}}{3} \qquad \rightarrow \qquad \dot{a} = H_{\mathcal{V}}a\,, \tag{30}$$

which has the solution

$$a \propto e^{H_{\mathcal{V}}t} = e^{\sqrt{\frac{8\pi G\mathcal{V}}{3}}t} = e^{\sqrt{\frac{8\pi G\rho_\phi}{3}}t}\,, \tag{31}$$

i.e., exponential growth of a, or in other words, inflation.

5 Putting the Brakes on Inflation

So, what stops the exponential explosion? We need to follow the cause through several of our previous equations.

1. Note from the figure of assumption 2, once the universe rolls off the peak $\frac{\partial\mathcal{V}}{\partial\phi} \neq 0$, so from (24), $\ddot{\phi}$, which was small initially, becomes large, meaning $\dot{\phi}$ increases rapidly.
2. Then, in (12) and (18), (26) is no longer true, as we can't ignore kinetic terms. $\rho_\phi \neq \mathcal{V}$ and $p_\phi \neq -\mathcal{V}$.
3. Since $\rho_\phi \neq -p_\phi$ in (7), $\dot{\rho} \neq 0$, i.e., ρ no longer constant.
4. Then, in (8), no longer do we have $\dot{a} = (\text{constant}) \times a$, but $\dot{a} = (\text{variable}) \times a$.
5. So we don't have the solution (31), and we don't have exponential expansion.

6 Notes

1. Through coupling of the inflaton ϕ to other particles, the kinetic energy of $\frac{1}{2}\dot{\phi}^2$ as inflation ends is transferred to those other particle types. The inflaton energy is converted to mass-energy of other particles. Then $\rho_{\text{other particles}} >> \rho_\phi$ and we get the R-W/Friedmann/Lemaitre usual model (not exponential expansion type mass energy). $\mathcal{V}$ is generally small at this point (it is down in the valley of the Mexican hat), so $T^{\mu\nu}$ is mostly other particles (or we would get tendency toward inflation again).
2. The density of baryons is reduced dramatically during inflation. But many new ones arise from the transmutation of inflaton kinetic energy to quark (and thus, baryon) mass. The same is true for leptons. Magnetic monopoles, presuming such things actually exist, are likewise diluted dramatically, but they are not replenished via interactions with inflatons, like standard model particles are. This may be the reason no monopoles have ever been seen − they are just too rare.

3. Inflation stretches wavelengths and so, cools the non-inflaton particles of the universe. But when the kinetic energy of the inflatons is passed on, via interactions, to other particles, the temperature of those other particles rises. This is known as the re-heating phase of the universe's evolution.

4. There is tendency to focus on $\mathcal{V}(\phi)$ having the form of the figure in assumption 2, i.e., Mexican hat with the potential like that seen for the Higgs in e/w symmetry breaking. However, other possible functions $\mathcal{V}(\phi)$ can work. One example is $\mathcal{V}(\phi) = Ae^{b\phi}$, which has a solution in (24) and (8) of $a \propto t^n$. Such a solution could explode rapidly with many of the same ramifications.

5. Inflation could commence when the primeval chaos has some patch where the particle density drops to where the $\mathcal{V}$ of the inflaton field dominates the stress-energy tensor. Statistical variation from location to location could yield various locations where inflation would occur. Each such inflated bubble would, due to the explosive nature of the expansion, be quite out of touch with other such bubbles.

6. We don't know what came before inflation. One often hears that the universe started from a spacetime singularity, but we don't know for sure, since inflation wiped out any history of what went before.

7. We actually have a form of inflation today in the accelerating expansion of the universe. Its cause, dark energy, appears to be like the inflaton in that it has constant mass-energy density and pressure equal to the negative of the mass-energy. However, dark energy density is many, many orders of magnitude less than that of the inflaton during inflation. So, though the same relation (31) governs both inflation as well as today's universal acceleration, that of the former is extremely high, while that of the latter is very small. (Note that recent observations suggest that dark energy density might vary slightly over long time periods, implying that it does not actually function as a cosmological constant. The jury is still out on this.)

H.W. Problem #2. Try, without looking at this document, to write down the key equations and steps leading to (31).

H.W. Problem #3. Read https://en.wikipedia.org/wiki/Cosmic_inflation .

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7 Present Status of Inflation

While most physicists working in the area believe inflation did, in fact, take place, a number, some of considerable renown, do not. Roger Penrose, Paul Steinhardt, and Neil Turok, for examples, favor a big bounce universe without inflation.

And there are issues in the details yet to be resolved, as Allen Guth and others readily admit. Research is ongoing, now 40^+ years after inflation theory's discovery.

8 Further Reading:

- Alan H. Guth & Paul J. Steinhardt, "The Inflationary Universe", Scientific American, May 1984.
- Andrei Linde, "The Self-Reproducing Inflationary Universe", Scientific American, November 1994.
- Scott Watson, "An Exposition on Inflationary Cosmology", WWWarticle, 2000. http://ned.ipac.caltech.edu/level5/Watson/Watson_contents.html
- Alan H. Guth, "The Inflationary Universe : The Quest for a New Theory of Cosmic Origins", Perseus Books,1998.

H.W. Problem #1. Deduce (15).

$$\hat{T}^{00} = \left(\partial^0\phi\right)\left(\partial^0\phi\right) - \eta^{00}\hat{\mathcal{L}} \tag{15}$$

From inserting (14) and (2)

$$\rho_\phi = \hat{T}^{00} = \frac{\dot{\phi}^2}{2} + \frac{\left(\nabla\phi\right)^2}{2} + \mathcal{V} = \tfrac{1}{2}\left(\partial^0\phi\right)\left(\partial^0\phi\right) + \tfrac{1}{2}\left(\partial^i\phi\right)\left(\partial^i\phi\right) + \mathcal{V} \tag{14}$$

$$\hat{\mathcal{L}}_\phi = \tfrac{1}{2}\phi^{,\mu}\phi_{,\nu} - \mathcal{V}\left(\phi\right) = \tfrac{1}{2}\eta_{\mu\nu}\phi^{,\mu}\phi^{,\nu} - \mathcal{V}\left(\phi\right) \qquad \eta_{\mu\nu} = \text{ Minkowski metric} \tag{2}$$

into (15)

$$\begin{gathered}\tfrac{1}{2}\phi^{,0}\phi^{,0} + \tfrac{1}{2}\phi^{,i}\phi^{,i} + \mathcal{V} = \phi^{,0}\phi^{,0} - \eta^{00}\left(\tfrac{1}{2}\eta_{\mu\nu}\phi^{,\mu}\phi^{,\nu} - \mathcal{V}\left(\phi\right)\right) = \phi^{,0}\phi^{,0} - (1)\left(\tfrac{1}{2}\phi^{,0}\phi^{,0} + \tfrac{1}{2}\phi^{,i}\phi^{,i} - \mathcal{V}\left(\phi\right)\right) \\ \tfrac{1}{2}\phi^{,0}\phi^{,0} + \tfrac{1}{2}\phi^{,i}\phi^{,i} + \mathcal{V} = \tfrac{1}{2}\phi^{,0}\phi^{,0} + \tfrac{1}{2}\phi^{,i}\phi^{,i} + \mathcal{V}\end{gathered} \tag{32}$$